

Royal Education society's
College of Computer Science and Information Technology, Latur.
Department of Computer Science

Academic Year (2022-23) class/semester: Bsc(CS) SY SEM-III

Name of Paper: MTCs (304-B) Prepared by : Mr. S.P. Bhosale.

Time 3 hr. Model Question Paper marks - 75

Q 1 Attempt any five of the following 15M

- Describe graph's.
- Describe Relation.
- Find HCF & LCM of 404 and 96
- Explain probability

e) find the determinant of A, $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

f) How many natural numbers are there between 23 and 100 which are exactly divisible by 6?

g) find Domain and Range, if $X = \{1, 2, 3, 4, 5\}$

$$Y = \{3, 6, 9, 12, 15\}, R = \{(x, y) / y = 3x\}$$

Q 2 Attempt any three of the following 15M

- Explain Degree of Vertices with example.
- If $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{4, 5, 6, 7, 8\}$ and universal set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ then prove that $(A \cup B) = (A - B) \cup (A \cap B) \cup (B - A)$
- Describe types of Relation.
- Find the following terms in geometric progression 3, 6, 12, 24, 48, ..., ..., 768.
- find AB, where.

$$A = \begin{bmatrix} -1 & -2 \\ -3 & 2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

Q.3 Attempt any three of the following 15M

a) Describe types of matrices

b) If

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 4 \\ -4 & 1 & 5 \end{bmatrix}$$

find the matrix 'X' such that

$A+X$ is unit matrix of order 3×3 .

c) A does a work in 4 days and B does the same work in 8 days. In how many days they together will do the same work.

d) Prove that;

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

e) Explain types of graph.

Q.4 Attempt any three of the following 15M

a) Describe arithmetic progression with example.

b) Explain sample space with example.

c) find the inverse of matrix A; Where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

d) If $A = \{a, e, i, o, u\}$, $B = \{b, d, f, l, m, s\}$, $C = \{i, o, m, n, w, z\}$ and $U = \{a, b, c, \dots, x, y, z\}$ then find.

i) $(A \cup C) \cap B$ (ii) $(A - B)$

iii) $(B - A)$ (iv) $(A' - B')$

e) If two dice are rolled simultaneously find probability of following event

i) The sum of digit on upper face is at least 10

ii) The digit on first die is greater than the digit on second die.

Q5

Attempt any three of the following

15 M

- a) Explain types of set
- b) A car moves at a speed of 108 km/hr, find the speed of the car in meter per second.
- c) Define event & explain its type.
- d) A bag contains 7 red and 4 white balls two balls are drawn at random. What is the probability that both the balls are red.
- e) Explain Isomorphism.

Model Answer key

Q 1 Attempt any five of the following

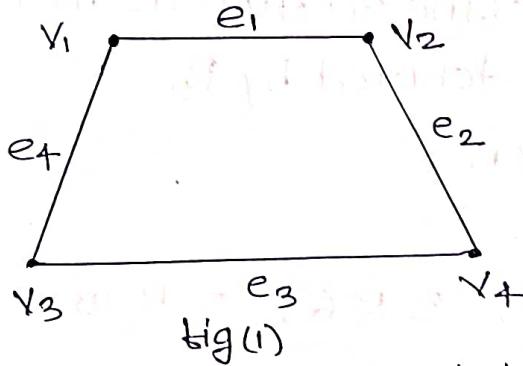
Q) Describe graph's.

→ A diagram in which a line or a curve shows the relationship between two quantities, measurements.
eg :- Income & expenditure, yearly sale of company.

A linear graph or a simple graph consist of
 $G = (V, E)$ consist of objects.

Where, $V = \{v_1, v_2, v_3, \dots, v_i\}$ is called vertices, &
 $E = \{e_1, e_2, e_3, \dots, e_k\}$ is called edges.

The vertices v_i associated with edge e_k are called
the end vertices of e_k .



The most common representation of a graph is by means of diagram, in which the vertices are represented as a point & each edge as line segment joining its end vertices.

Above figure shows the four vertices & four edges.

Q.1.b) Describe Relation.?

→ The concept of term relation is drawn from the meaning of a relation in English language, According to which two quantities or objects are related to each other, if there is recognizable link b/w them.

Members in family are often related to each other. We describe relation as A is friend of B, or p. is sister of q etc. In mathematics we have different relation.

A well defined relation between elements of A and B is given by $a R b$, where $a \in A$ & $b \in B$. Thus relation gives ordered pair (a, b) & defines a subset $A \times B$.

Eg:- We can define relation as 'm' is factor of 'n'. If this relation denoted by R,

so. $m R n$

then we write.

$2 R 4, 3 R 6, 5 R 10$

Q.1.c) Find HCF of 404 and 96.

→ first we have to find out the factors of a number 404 & 96.

∴ so.

$$\begin{array}{r|rr} 2 & 404 \\ \hline 2 & 202 \\ \hline 101 & 101 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|rr} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$404 = 2 \times 2 \times 101$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$\therefore \text{HCF} = 2 \times 2 \\ = 4.$$

\therefore the HCF of 404 & 96 is 4.

Q1.d) Explain Probability?

→ A random experiment poses uncertainty regarding the actual result of the experiment, even though all possible outcomes are already known. The classical definition of probability is based on the assumption that all possible outcomes of an experiment are equally likely.

Eg:- i) possibly, it will rain tonight.

ii) there is high chance of getting the result next month.

In mathematical language, when possibility of an expected event is expressed in number, it is called as probability.

- It is expressed as a fraction or percentage using the following formula,

The probability of an event A is defined as

$$P(A) = \frac{n(A)}{n(S)}$$

$\therefore n(A)$ = Number of favourable outcomes

$n(S)$ = Number of all possible outcomes.

Q.1.e) Find the determinant of A, $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

→ Let, The given matrix is.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

Determinant of matrix A,

$$\begin{aligned} |A| &= 1[(3 \times 1) - (-2 \times 0)] - 2[(-1 \times 1) - (0)] + (-2)[(-1 \times -2) - (0)] \\ &= 1[3 - 0] - 2[-1 - 0] - 2[2 - 0] \end{aligned}$$

$$= 3 - 2(-1) - 2(2)$$

$$= 3 + 2 - 4$$

$$\therefore |A| = 1$$

Q.1.f) How many natural numbers are there between 23 and 100 which are exactly divisible by 6?

→ The numbers exactly divisible by 6 are 24, 30, 36, 42...

∴ The difference between two consecutive numbers is 6. so the sequence is in arithmetic progression

(i) Find first and last term of sequence 23, 27, 31, ..., 96.

∴ By using arithmetic progression formula .

$$a_n = a + (n-1) \cdot d,$$

where, $a_n = n^{\text{th}}$ term which is '96'

a = initial term which is 24

n = Number of terms

d = common difference which is '6'

putting all values in above formula.

$$a_n = a + (n-1) \cdot d$$

$$96 = 24 + (n-1) \cdot 6$$

solving further,

$$(n-1) \times 6 = 96 - 24$$

$$(n-1) = \frac{72}{6} = 12$$

$$(n-1) = 12$$

$$n = 12 + 1$$

$$\boxed{n = 13}$$

∴ There are 13 natural number between 23 and 96 which are divisible by 6.

Q1

g) find Domain and Range.

$$\text{If } X = \{1, 2, 3, 4, 5\}, Y = \{3, 6, 9, 12, 16\}, R = \{(x, y) | y = 3x\}$$

→ let the given sets

$$X = \{1, 2, 3, 4, 5\}$$

$$Y = \{3, 6, 9, 12, 16\}$$

the given relation,

$$R = \{(x, y) | y = 3x\}$$

$$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

∴ Domain is first component of set

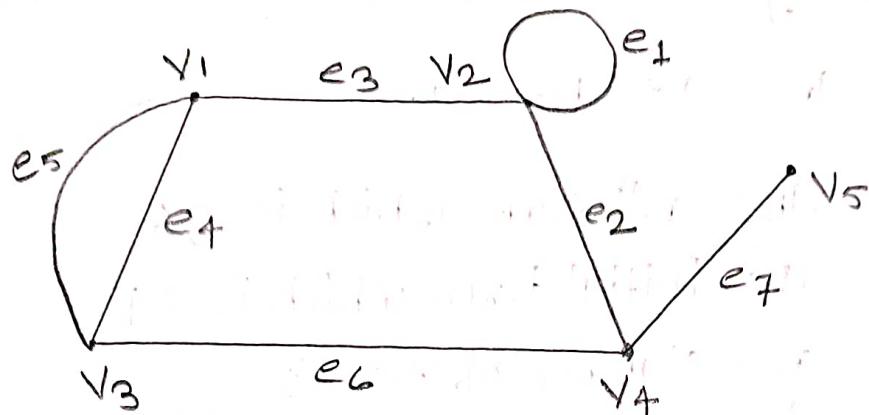
$$\therefore \text{Domain} = \{1, 2, 3, 4\}$$

Range is second component of set

$$\therefore \text{Range} = \{3, 6, 9, 12\}$$

- Q 2 Attempt any three of the following
- a) Explain Degree of Vertices with example.
- The number of edges incident on a vertex (v_i), with self loop counted twice is called degree of vertex.

Degree of vertex is denoted by $d(v)$



In the above figure,

$$d(v_1) = 3$$

$$d(v_2) = 4$$

$$d(v_3) = 3$$

$$d(v_4) = 3$$

$$d(v_5) = 1$$

Now consider

$$\begin{aligned} \text{Total degree of figure} &= d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5) \\ &= 3 + 4 + 3 + 3 + 1 \\ &= 14 \end{aligned}$$

which is equal to twice the number of edges

$$\begin{aligned} \text{i.e. } \sum_{i=1}^n d(v_i) &= 2 \cdot e \\ &= 2 \times 7 \\ &= 14. \end{aligned}$$

Q. 2 b) If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8\}$
and Universal set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ then
prove that $(A \cup B) = (A - B) \cup (A \cap B) \cup (B - A)$

→ let the given sets are

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

$$C = \{4, 5, 6, 7, 8\}$$

$$X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

∴ the eqn is $(A \cup B) = (A - B) \cup (A \cap B) \cup (B - A)$

$$\therefore LHS = (A \cup B)$$

$$= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6\} \quad \text{--- (i)}$$

similarly

$$RHS = (A - B) \cup (A \cap B) \cup (B - A)$$

$$\therefore A - B = \{1, 2, 3, 4\} - \{3, 4, 5, 6\}$$

$$= \{1, 2\}$$

$$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$$

$$= \{3, 4\}$$

$$B - A = \{3, 4, 5, 6\} - \{1, 2, 3, 4\}$$

$$= \{5, 6\}$$

$$\therefore RHS = (A - B) \cup (A \cap B) \cup (B - A)$$

$$= \{1, 2\} \cup \{3, 4\} \cup \{5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6\} \quad \text{--- (ii)}$$

From eqn (i) & (ii),

$$LHS = RHS$$

$$\therefore (A \cup B) = (A - B) \cup (A \cap B) \cup (B - A)$$

Hence Proved.

Q.2.c) Describe types of relation:

→ The following are the types of relation.

i) Reflexive Relation:

A relation R on a set A is reflexive if aRa for every $a \in A$.

i.e. if $(a, a) \in R$ for every $a \in A$

Let R be a binary relation on A , R is said to be reflexive relation if (a, a) is in R for every a in A .

In other words in a reflexive relation every element in A is related to itself.

e.g.: If $A = \{1, 2, 3\}$

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$

ii) Symmetric Relation:

Let R be a binary relation on A , R is said to be symmetric relation if (a, b) is in $R \Rightarrow (b, a)$ is also in R .

A Relation R on a set A is symmetric if whenever aRb implies that bRa .

i.e. if whenever $(a, b) \in R$ then $(b, a) \in R$

e.g.: If $A = \{1, 2, 3\}$

$$R_1 = \{(1, 1), (1, 2), (2, 1)\}$$

R_1 is a symmetric relation

iii) Transitive Relation:

A relation R on A is said to be transitive if whenever aRb and bRc then aRc .

i.e. if whenever $(a, b), (b, c) \in R$ then $(a, c) \in R$

e.g. - Let $A = \{a, b, c\}$ &

$$R_1 = \{(a, a), (a, b), (a, c), (b, c)\}$$

∴ R is transitive relation.

Q.2 d) Find the following terms in geometric progression

3, 6, 12, 24, 48, ..., ..., ..., 768.

→ The ratio of two consecutive numbers is same so,
∴ the given sequence is in geometric progression.

∴ formula for geometric progression is

$$a_n = a \cdot r^{n-1}$$

where $a_n = n^{\text{th}}$ term

a = Initial term

r = common ratio

n = number of terms

∴ given sequence is

3, 6, 12, 24, 48, ..., ..., ..., 768.

∴ We have to find 6th, 7th & 8th term.

$$\begin{aligned}a_6 &= 3 \cdot (2)^{6-1} \\&= 3 \times 2^5 \\&= 3 \times 2 \times 2 \times 2 \times 2 \times 2 \\&= 96\end{aligned}$$

similarly, $a_7 = 3 \cdot (2)^{7-1}$

$$\begin{aligned}&= 3 \times 2^6 \\&= 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\&= 192.\end{aligned}$$

& lastly, $a_8 = 3 \cdot (2)^{8-1}$

$$\begin{aligned}&= 3 \times 2^7 \\&= 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\&= 384.\end{aligned}$$

∴ the series will be. 3, 6, 12, 24, 48, 96, 192, 384, 768.

$$\text{Q.2.e)} \text{ Find } AB. \text{ Where, } A = \begin{bmatrix} -1 & -2 \\ -3 & 2 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

→ let, given matrix are.

$$A = \begin{bmatrix} -1 & -2 \\ -3 & 2 \\ 1 & 0 \end{bmatrix}_{3 \times 2}, B = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}_{2 \times 2}$$

If the number of columns in matrix A should equal to number of rows in matrix B.

$$A \cdot B = \begin{bmatrix} -1 & -2 \\ -3 & 2 \\ 1 & 0 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}_{2 \times 2}$$

$$= A_{3 \times 2} \times B_{2 \times 2}$$

$$= \begin{bmatrix} -1 + (-2x-1) & -2 + (-2x-2) \\ (-3x1) + (2x-1) & (-3x2) + (2x-2) \\ (1x1) + 0 & (1x2) + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & -2+4 \\ -3-2 & -6-4 \\ 1+0 & 2-0 \end{bmatrix}$$

$$\therefore A \cdot B = \begin{bmatrix} 1 & 2 \\ -5 & -10 \\ 1 & 2 \end{bmatrix}_{3 \times 2}$$

Q.3) Attempt any three of the following

c) Describe types of matrices.

→ there are several types of matrices

i) Row Matrix :- A matrix having only one row is called a row matrix.

- It is order $1 \times n$.

e.g:- $\begin{bmatrix} -1 & 2 \end{bmatrix}_{1 \times 2}$

ii) $\begin{bmatrix} 0, 2 & -4 & 5 \end{bmatrix}_{1 \times 4}$

iii) Column Matrix :- A matrix having only one column is called column matrix.

- It is order $m \times 1$.

e.g:- $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} b \\ d \\ a \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3 \times 3}$

iv) Square Matrix :- A matrix with equal number of rows and columns is called square matrix.

- It is order of $m \times m / n \times n$

e.g:- $\begin{bmatrix} -1 & 3 \\ 5 & 7 \end{bmatrix}_{2 \times 2}, \begin{bmatrix} 2 & 3 & 7 \\ 4 & 5 & -3 \\ 8 & 1 & 6 \end{bmatrix}_{3 \times 3}$

v) Scalar Matrix :- A diagonal matrix in which all the diagonal elements are same is called scalar matrix.

e.g:- $A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}_{3 \times 3}, B = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}_{2 \times 2}$

vi) A Diagonal Matrix :- A square matrix in which all non-diagonal elements is zero is called a diagonal matrix.

e.g:- $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3 \times 3}, B = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}_{2 \times 2}$

Q.8. If $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 4 \\ -4 & 1 & 5 \end{bmatrix}$ find the matrix 'X' such that

$\rightarrow A + X$ is a unit matrix of order 3×3 .

let. the given eqn is

$$A + X = \text{Unit Matrix.}$$

$$\therefore \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 4 \\ -4 & 1 & 5 \end{bmatrix} + X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We have to find out the matrix 'X'

$$\therefore X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 4 \\ -4 & 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 0-(-1) & 0-0 \\ 0-3 & 1-2 & 0-4 \\ 0-(-4) & 0-1 & 1-5 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & 1 & 0 \\ -3 & -1 & -4 \\ 4 & -1 & -4 \end{bmatrix}$$

\therefore the matrix 'X' will be

$$X = \begin{bmatrix} -1 & 1 & 0 \\ -3 & -1 & -4 \\ 4 & -1 & -4 \end{bmatrix}$$

Q. 3

c) A does a work in 4 days & B does the same work in 8 days. In how many days they together will do the same work?

→ A does the work in 4 days

∴ A's 1 day work will be $\frac{1}{4}$ days
similarly,

B does the work in 8 days

so, B's 1 day work will be $\frac{1}{8}$ days

∴ together they will finish the same work will be equal to 'A' one day work in addition with 'B's one day work.

i.e $A+B =$ 'A's 1 day work + B's 1 day work

$$= \frac{1}{4} + \frac{1}{8}$$

solving further,

$$= \frac{12}{32} = \frac{3}{8}$$

$(A+B)$'s 1 day work = $\frac{3}{8}$ days

∴ Total days required to finish total work while working together = $\frac{8}{3}$ days.

Q3 d) Prove that, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



→ The given eqⁿ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

let us consider sets

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

$$C = \{4, 5, 6, 7, 8\}$$

$$LHS = A \cup (B \cap C)$$

$$\therefore B \cap C = \{3, 4, 5, 6\} \cap \{4, 5, 6, 7, 8\}$$

$$B \cap C = \{4, 5, 6\}$$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6\} \quad (i)$$

Now, $RHS = (A \cup B) \cap (A \cup C)$

$$\therefore (A \cup B) = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$$
$$= \{1, 2, 3, 4, 5, 6\}$$

$$(A \cup C) = \{1, 2, 3, 4\} \cup \{4, 5, 6, 7, 8\}$$
$$= \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$\therefore (A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5, 6, 7, 8\}$$
$$= \{1, 2, 3, 4, 5, 6\} \quad (ii)$$

from eqⁿ (i) & (ii)

$$LHS = RHS.$$

∴ Hence

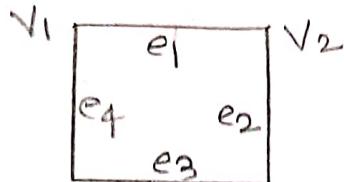
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Hence proved.

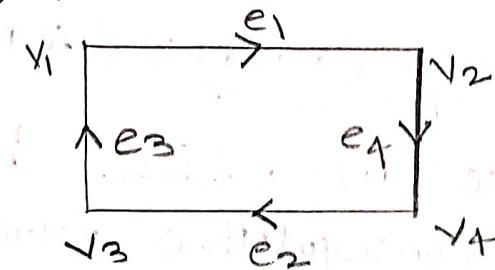
Q3. e) Explain types of graph.

→ Various important types of graph in graph theory are

i) Undirected Graph:- A graph in which all the edges are undirected is called undirected graph.



ii) Directed Graph:- If each edge of the graph has a direction then the graph is called directed graph.



Here all the edges have direction.

iii) Null graph:- A graph which contains only isolated vertices or nodes is called a null graph.

The set of edges in null graph is empty

A . . B

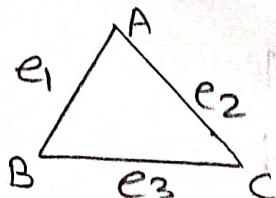
D . . C

iv) Trivial Graph:- A graph having only one vertex in it is called trivial graph.

Eg:- A .

• It is smallest possible graph

v) Finite Graph:- A graph $G(V, E)$ consisting of finite number of vertices & edges is called as finite graph



Graph contain three vertex & three edges.

Q4 Attempt any THREE of the following (5M each)

a) Describe arithmetic Progression with examples.

* Sequence: The arrangement of numbers in definite order is called as sequence.

* Arithmetic Progression (A.P.)

The sequence in which difference b/w any two consecutive number is constant is called as Arithmetic progression.

for ex:- $N = 1, 2, 3, 4, 5, \dots \rightarrow \infty$
AP. → Perfect square no.

No. AP: - 1, 4, 9, 16, 25, ..., ∞
Perfect cube no.

No. AP 1, 8, 27, 64, 128, ..., ∞

* Sum of A.P.

$$S_n = \frac{n}{2} (a + l)$$

or

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

* nth term of A.P

$$t_n = a + (n-1)d$$

Ex.

1) Find the sum of first 100 natural nos.

⇒ These numbers are,

1, 2, 3, 4, ..., 100

This is a A.P.

$$\therefore S_n = \frac{n}{2} (a + l)$$

$$S_n = \frac{100}{2} (1 + 100)$$

$$S_n = 5050 (\text{Ans})$$

$$\boxed{S_n = 5050}$$

∴ Sum of first 100 natural nos is 5050

2) How many natural numbers are there between 20 to 40 which are exactly divisible by 3.

⇒ These nos are 21, 24, 27, 30, 33, 36, 39

$$\therefore t_n = a + (n-1)d$$

$$\text{WKT, } t_n = 39, \quad a = 21, \quad d = 3$$

$$39 = 21 + (n-1)$$

$$39 - 21 = 3n - 3$$

$$18 + 3 = 3n$$

$$\frac{21}{3} = n$$

$$\boxed{n = 7}$$

Q4 b) Explain sample space with examples ?

- The set of all possible outcomes of a random experiment is called the sample space .
- The sample space is denoted by 'S' or Ω (omega)
 - The Number of sample space is denoted by $n(S)$.

Eg:- Random experiment & sample space

i) One coin is tossed

$$S = \{H, T\}$$

Number of sample space .

$$\therefore n(S) = 2$$

ii) Two coins are tossed

$$S = \{HH, HT, TH, TT\}$$

∴ Number of sample space .

$$\therefore n(S) = 4$$

iii) A die is thrown .

$$\therefore S = \{1, 2, 3, 4, 5, 6\}$$

Number of sample spaces

$$\therefore n(S) = 6$$

iv) If three coins are tossed .

$$\therefore S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

∴ Number of sample spaces .

$$n(S) = 8.$$

Q4.c) find the inverse of matrix A, where, $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

→ The given matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

if $|A| \neq 0$, then only we will find the inverse of the matrix.

So, first we have to find $|A|$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$$

$$|A| = (1 \times 7) - (2 \times 3)$$

$$= 7 - 6$$

$$= 1$$

$$\therefore |A| = 1$$

$\therefore |A| \neq 0$, Inverse is possible

$$\therefore A^{-1} = \frac{\text{Adj. of matrix } A}{|A|}$$

So first we have to find out the Adjoint matrix 'A'

$$\text{Adj. of matrix } A = [\text{co-factor matrix}]^T$$

∴ let's find co-factor matrix.

co-factor matrix.

$$A_{11}(1) = (-1)^{1+1} (7) = (-1)^2 (7)$$

$$= 1 \times 7 = 7$$

$$= 7$$

$$A_{12}(2) = (-1)^{1+2} (3) = (-1)^3 \cdot (3)$$

$$= (-1) \times 3$$

$$= -3$$

$$\begin{aligned}
 A_{21} &= (-1)^{2+1} (2) \\
 &= (-1)^3 (2) \\
 &= (-1) \times 2 \\
 &= -2
 \end{aligned}$$

$$\begin{aligned}
 A_{22} &= (-1)^{2+2} (1) \\
 &= 1 \times 1 \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Cofactor matrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \\
 &= \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Adjoint of matrix } A &= [\text{Cofactor matrix}]^T \\
 &= \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}^T
 \end{aligned}$$

$$\text{Adj. of } A = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj. of } A}{|A|}$$

$$= \frac{1}{1} \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\therefore \text{Inverse of matrix } A = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

Q4-d) If $A = \{a, e, i, o, u\}$, $B = \{b, d, f, l, m, s\}$, $C = \{j, o, m, n, w, z\}$

and $U = \{a, b, c, \dots, x, y, z\}$ then find.

i) $(A \cup C) \cap B$ ii) $(A - B)$ iii) $(B - A)$ iv) $(A' - B')$

→ Let. the given sets are.

$$A = \{a, e, i, o, u\}$$

$$B = \{b, d, f, l, m, s\}$$

$$C = \{j, o, m, n, w, z\} \text{ &}$$

$$U = \{a, b, c, \dots, x, y, z\}$$

i) $(A \cup C) \cap B$

$$\therefore (A \cup C) = \{a, e, i, o, u\} \cup \{j, o, m, n, w, z\}$$
$$= \{a, e, i, o, u, m, n, w, z\}$$

$$(A \cup C) \cap B = \{a, e, i, o, u, m, n, w, z\} \cap \{b, d, f, l, m, s\}$$

$$(A \cup C) \cap B = \{m\}$$

ii) $(A - B) = \{a, e, i, o, u\} - \{b, d, f, l, m, s\}$

$$A - B = \{a, e, i, o, u\}$$

iii) $(B - A) = \{b, d, f, l, m, s\} - \{a, e, i, o, u\}$

$$(B - A) = \{b, d, f, l, m, s\}$$

iv) $(A' - B') =$

$$\therefore U = \{a, b, c, \dots, x, y, z\}$$

$$A' = \{a, e, i, o, u\}$$

$$A' = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$$

$$B' = \{a, c, e, g, h, i, j, k, m, o, p, q, r, t, u, v, w, x, y, z\}$$

$$\therefore A' - B' = \{b, d, f, l, m, s\}$$

Q4) If two dice are rolled simultaneously find the probability of the following event.

- i) The sum of digit on the upper face is atleast 10
ii) The digit on the first die is greater than the digit on second die.

→ When two dice are rolled,

let. S be the sample space.

$$\therefore S = \{(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) \\ (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) \\ (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) \\ (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) \\ (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) \\ (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)\}$$

∴ Total number of sample space

$$\text{i.e } n(S) = 36.$$

- i) sum of digits on upper face is atleast 10

$$\text{i.e. } A = \{(4,6) (5,5) (5,6) (6,4) (6,5) (6,6)\}$$

$$\therefore n(A) = 6.$$

$$\text{Probability } P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(A) = \frac{1}{6}$$

- ii) The digit on the first die is greater than the digit on second die.

$$\text{i.e } A = \{(2,1) (3,1) (3,2) (4,1) (4,2) (4,3) (5,1) (5,2) \\ (5,3) (5,4) (6,1) (6,2) (6,3) (6,4) (6,5)\}$$

$$\therefore n(A) = 15$$

$$\therefore \text{Probability of } A. \text{ i.e } P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{15}{36}$$

By solving further.

$$\therefore P(A) = \frac{5}{12}$$

∴ probability of digit on the first die is greater than the digit on second die. is $\frac{5}{12}$

Q5

Attempt any three of the following.

15

a) Explain types of set?

→ set :- A collection of well defined object is called as set

∴ there are several types of set

i) Empty set :- the set containing no element is called as empty set

• It is also called as null set or void set

• It is denoted by symbol \emptyset or {}

eg :- Natural numbers less than 1

∴ {}

ii) Singleton set :- A set containing only one element is called singleton set

eg:- let A be the set of all integers which are neither positive nor negative.

∴ $A = \{0\}$

iii) finite set :- The empty set or set which contain finite number of elements is called as finite set.

eg:- set of letters in word 'Mahesh'

∴ $A = \{M, A, H, E, S, H\}$

$n(A) = 6$

iv) Infinite set :- A set which is not finite is called an infinite set.

Eg:- set of natural numbers
set of rational numbers

v) Equal Set :- Two sets are said to equal if they contain the same elements.

i.e. if $A \subseteq B$ and $B \subseteq A$

Eg:- let X be the set of letters in word ABBA and Y be the set of letters in word BABA

$$\therefore X = \{A, B\}$$

$$Y = \{B, A\}$$

\therefore The sets X & Y are equal sets.

Q5

b) A car moves at a speed of 108 km/hr, find the speed of a car in meter per second?

→

Let, the given speed of car is 108 km/hr

\therefore So, we required speed in m/s

$$\therefore 1 \text{ km} = 1000 \text{ meters}$$

Similarly, we have to convert hours into seconds

$$\therefore 1 \text{ hour} = 60 \text{ minutes}$$

$$\therefore 1 \text{ hour} = 60 \times 60 \text{ seconds}$$

\therefore Now we will convert km/hr into m/s.

$$\therefore 1 \text{ km/hr} = \frac{1000}{60 \times 60} \text{ m/sec}$$

By solving further,

$$1 \text{ km/hr} = \frac{\frac{5}{18}}{3600} \text{ m/s}$$

$$\therefore 1 \text{ km/hr} = \frac{5}{18} \text{ m/s.}$$

$$\therefore 108 \text{ km/hr} = 108 \times \frac{5}{18} \text{ m/s}$$

$$= 6 \times 5 \text{ m/s}$$

$$108 \text{ km/hr} = 30 \text{ m/s.}$$

Q5

C) Define event & Explain its type?

→ Event :- A set of favourable outcomes of a given sample space is called an event.

- Event is subset of sample space.

- Events are denoted by capital letters A, B, ...

Eg:- If two coins are tossed & A is the event of getting atleast one Head.

$$\therefore S = \{HH, HT, TH, TT\}$$

$$A = \{HH, HT, TH\}$$

$$\therefore n(A) = 3$$

There are several types of events, as follows

i) Elementary Event :- An event consist of single outcomes is called as elementary event

- It is also called as simple event

Eg:- Tossing a coin & getting only Head. Page 13

ii) Certain Event :- The sample space is called the certain event if all possible outcomes are favourable outcomes i.e. event consists of whole sample space.

iii) Impossible Event :- the empty space is called impossible event as no possible outcome is favourable

Eg:- Throwing a single die

$$\therefore S = \{1, 2, 3, 4, 5, 6\} \text{ &}$$

event is Number greater than '6'.

Q5

d) A bag contains 7 red & 4 white balls. Two balls are drawn at random. What is probability that both the balls are red.

→

Given that

Total number of red ball's are 7 and total

Number of white balls are 4

also given condition is both selected balls should be red

∴ Total number of balls are 11

∴ Number of balls are selected = ${}^{11}C_2$

& possibility of both the balls are red

will be 7C_2

∴ so the probability of selected balls are red will be

$$\therefore \text{probability} = \frac{7C_2}{11C_2}$$

$$\begin{aligned}\therefore 7C_2 &= \frac{7!}{2!(7-2)!} = \frac{7!}{2! \times 5!} \\ &= \frac{7 \times 6 \times 5!}{2 \times 1 \times 5!} \\ &= 21\end{aligned}$$

$$\begin{aligned}11C_2 &= \frac{11!}{2!(11-2)!} = \frac{11!}{2! \times 9!} \\ &= \frac{11 \times 10 \times 9!}{2 \times 1 \times 9!} \\ &= 55\end{aligned}$$

$$\therefore \text{Probability} = \frac{7C_2}{11C_2} = \frac{21}{55}.$$

Q5) Explain Isomorphism of graphs.

Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be isomorphic if there exists a function $f: V_1 \rightarrow V_2$ such that

i) f is one to one onto i.e. f is bijective.

ii) $\{a, b\}$ is an edge in E_1 , if $\{f(a), f(b)\}$ is an edge in E_2 for any two numbers $a, b \in V_1$.

Any function ' f ' with above properties is called an isomorphism in G_1 & G_2 .

e.g:-

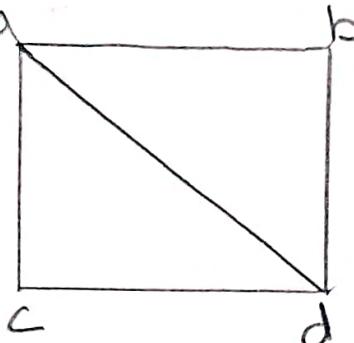


Fig (1)

$G_1(V_1, E_1)$

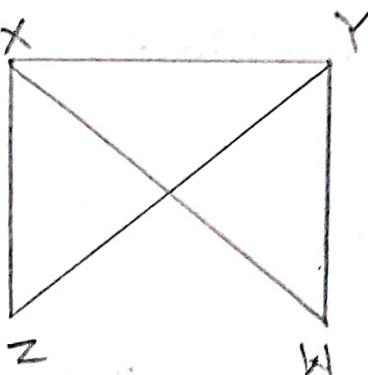


Fig (2)

$G_2(V_2, E_2)$

We have to check the given figures are isomorphic

1) Number of vertex

Fig (1)

4

Fig (2)

4

2) Number of edges

5

5

3) Degree sequence

a - 3

x - 3

b - 2

y - 3

c - 2

z - 2

d - 3

w - 2

4) Mapping :- by mapping we have to check the degrees of each individual vertex.

If $G_1(V_1, E_1)$ & $G_2(V_2, E_2)$ are isomorphic graphs

then G_1 and G_2 have the

$$|V_1| = |V_2|$$

$$|E_1| = |E_2|$$

\therefore If any of these qualities differ in the graph, then they cannot be isomorphic.